STAFF SUMMARY SHEET

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1	DFP	sig	Timothy W Russell, O5 11 June 2014	6			
2	DFER	approve	Soltz, AD22, 12 Jun 14	7			
3	DFP	action		8			
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SUMMARY

1. PURPOSE. To provide security and policy review on the document at Tab 1 prior to release to the public.

2. BACKGROUND.
Author: Lt Col Steven Novotny x9248

Title: Inverse Trilateration - A Positional Goal Seeking Algorithm for a Swarm of Autonomous Drones

Document type: Journal Article

Description: In this paper the author describes a solution to the reverse GPS problem and its application to positional goal seeking for a swarm of autonomous drones. The autonomous objects, using estimates of their own relative (not absolute) locations, estimated ranges to one another, and range data to a goal will converge on that goal even in a GPS-denied environment.

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3. DISCUSSION. None.

4. VIEWS OF OTHERS. None.

5. RECOMMENDATION. Sign coord block above indicating document is suitable for public release. Suitability is based solely on the document being unclassified, not jeopardizing DoD interests, and accurately portraying official policy.

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Director of Research
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Tabs
1. Document

Inverse Trilateration - A Positional Goal Seeking Algorithm for a Swarm of Autonomous Drones

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Abstract

In this paper I describe a solution to the reverse GPS problem and its application to positional goal seeking for a swarm of autonomous drones. GPS uses known positions and corresponding ranges from multiple beacons to a single location to determine the coordinates of that location. In contrast, this work uses range measurements from a single target or beacon (of unknown position) to a group of cross-communicating autonomous objects. These autonomous objects, using estimates of their own relative (not absolute) locations, estimated ranges to one another, and range data to the goal will converge on that goal. The convergence algorithm is based on a linear least-squares approach thus creating minimal demands for on-board computational resources. An example application is used to demonstrate the robustness of the approach by using a simple range finder based on assumed transmitter power and a simple attenuation model. The results show the algorithm and convergence are not sensitive to errors in the assumed attenuation.

Keywords: Autonomous drones, inverse trilateration, least squares optimization, quadrotor

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1. Inverse Trilateration

The Global Positioning System (GPS) uses a geometric process called trilateration to determine the position of a receiver. Most simply stated, trilateration is a process of determining a position by considering the intersection of multiple spheres. The minimum number of spheres required to determine a three dimensional position is four. The intersection of two spheres (assuming they intersect) will create a circle; the third is required to reduce the set of possible points to two; and the fourth reduces the set to a unique point.

The GPS uses a sophisticated constellation of satellites that transmit position and timing data continuously. Each satellite carries an atomic clock and regularly updated ephemeris data. The GPS has proven itself immensely beneficial in innumerable civilian and military applications. In ideal circumstances, positions for all objects at all times can be determined and known with high accuracy through GPS. In ideal circumstances, the work described in this paper would be unnecessary.

In many situations GPS is unavailable or unreliable. This condition can be caused by geographical obstacles such as those encountered in mountainous terrain. The condition may also occur through man-made effects, such as intentionally-induced GPS denied environments. In these situations, alternative methods must be invoked–ones that do not rely upon absolute position.

The problem described in this paper is different from other work on navigating in a GPS-denied environment, such as Simultaneous Localization and Mapping or SLAM. SLAM is the process of sensing an unknown environment using available sensor data to develop both the pose of the drone and a map of the environment [1]. This work is concerned only with goal seeking: finding the simplest way for a swarm of autonomous drones to converge upon an object with no requirement for absolute position.

1.1. The Reverse GPS Problem

The reverse GPS problem considers a single beacon transmitting a signal from which a range (but not position) can be determined. These range mea-

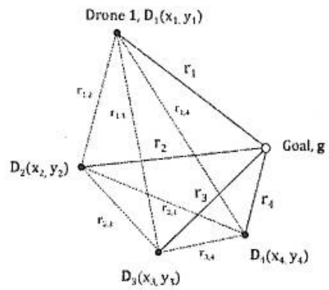


Figure 1: Geometry for a group of four drones and a single (goal) beacon.

surements can then be used by a group of autonomous entities who share data about their own estimated position relative to one another and their estimated range to the beacon. Figure 1 illustrates the geometry of the problem, with the real world 3D problem reduced to 2D for simplicity.

The objective for each drone is to determine a velocity vector which will move it from its present location towards the goal. The necessary velocity vector for each drone is determined through the following:

1. its own estimated position, (x_i, x_j)
2. its own estimated range to the goal beacon, r_i
3. the estimated positions of all other drones in the swarm, (x_j, y_j)
4. the estimated range of each drone to the goal beacon, r_j

Based on figure 1 the distance between drone i and drone j is given by

$$r_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{1}$$

The distance between drone i and the goal is given by

$$r_i = \sqrt{(g_x - x_i)^2 + (g_y - y_i)^2} \tag{2}$$

Taking the above equation and mixing in the influence of the other drones (by adding and subtracting x_j and y_j) gives

$$\begin{aligned} r_i^2 &= (g_x - x_i)^2 + (g_y - y_i)^2 \\ &= (g_x - x_j + x_j - x_i)^2 + (g_y - y_j + y_j - y_i)^2 \\ &= (g_x - x_j)^2 + 2(g_x - x_j)(x_j - x_i) + (x_j - x_i)^2 \\ &\quad + (g_y - y_j)^2 + 2(g_y - y_j)(y_j - y_i) + (y_j - y_i)^2 \end{aligned} \tag{3}$$

Rearranging terms gives

$$\begin{aligned} &-2[(g_x - x_j)(x_i - x_j) + (g_y - y_j)(y_i - y_j)] = \\ &\qquad r_i^2 - [(g_x - x_j)^2 + (g_y - y_j)^2] - [(x_j - x_i)^2 + (y_j - y_i)^2] \end{aligned} \tag{4}$$

or

$$(g_x - x_j)(x_i - x_j) + (g_y - y_j)(y_i - y_j) = \frac{1}{2}(r_j^2 + r_{i,j}^2 - r_i^2) \tag{5}$$

If we now choose any one of the n drones, we can develop a system of equations for that particular drone–the one which will be attempting to find a solution. As such, equation 5 represents a set of $n-1$ linear equations for drone i. For the case of drone 1, equation 5 becomes

$$\begin{aligned} (g_x - x_1)(x_2 - x_1) + (g_y - y_1)(y_2 - y_1) &= \frac{1}{2}(r_1^2 + r_{1,2}^2 - r_2^2) \\ (g_x - x_1)(x_3 - x_1) + (g_y - y_1)(y_3 - y_1) &= \frac{1}{2}(r_1^2 + r_{1,3}^2 - r_3^2) \\ &\vdots \\ (g_x - x_1)(x_n - x_1) + (g_y - y_1)(y_n - y_1) &= \frac{1}{2}(r_1^2 + r_{1,n}^2 - r_n^2) \end{aligned} \tag{6}$$

Optimization techniques for improving GPS accuracy have been studied through least squares and non-linear least squares approaches using the same formulation given above [2]. However, the goal in the previous work was to find an absolute 3D position, i.e. g_x and g_y (and g_z). The goal in this work is to find the difference in positions between the unknown beacon and each individual drone, i.e. $(g_x - x_i)$ and $(g_y - y_i)$, from which drone commands can be generated.

1.2. Least Squares Solution

The set of equations developed in the previous section represent an overdetermined, linear system of equations. Because the distances r_i and $r_{i,j}$ and all positions are only approximate the problem lends itself to a least squares solution. For this problem, I will express the set of equations with the form $\mathbf{A} \cdot \mathbf{X} - \mathbf{b} \approx \mathbf{0}$ where

$$\mathbf{A} = \begin{pmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \\ \vdots & \vdots \\ x_n - x_1 & y_n - y_1 \end{pmatrix} \quad \mathbf{X} = \begin{pmatrix} g_x - x_1 \\ g_y - y_1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} \frac{1}{2}(r_1^2 + r_{1,2}^2 - r_2^2) \\ \frac{1}{2}(r_1^2 + r_{1,2}^2 - r_2^2) \\ \vdots \\ \frac{1}{2}(r_1^2 + r_{1,n}^2 - r_n^2) \end{pmatrix} \tag{7}$$

The least squares approach requires us to minimize the sum of the squares of the residuals, i.e

$$\nabla F(\mathbf{X}) = \mathbf{0} \tag{8}$$

where $F(\mathbf{X}) = \|\mathbf{AX} - \mathbf{b}\|^2 = (\mathbf{AX} - \mathbf{b})^{\mathrm{T}}(\mathbf{AX} - \mathbf{b})$. This leads to a solution in the form of

$$\mathbf{X} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b} \tag{9}$$

2. Simulations of an n-drone system using least squares optimization

Each drone was modeled as a single entity, with the ability to transmit to other drones its own estimated position and its measured range to the beacon. Each drone uses these values to find a solution from equation 9. The solution is converted to a velocity (or command) by normalizing the difference vector $\mathbf{X}$, i.e. $\mathbf{v} = \mathbf{X}/\|\mathbf{X}\|$ and assuming a time step of $\Delta t = 1$. Each drone object then advances using a simple proportional controller.

n drones	$\mathbf{X} = (g_x - x_1, g_y - y_1)$ (m)	σ_x (m)	σ_y (m)	$\|\sigma\|$
5	(962, 992)	510.8	331.2	608.7
10	(969, 993)	248.4	171.9	302.1
15	(987, 987)	145.0	147.8	210.6
20	(984, 998)	131.7	106.7	169.5

Table 1: Least squares solution for drone 1 as a function of n, the number of drones in the swarm. The beacon is located at a position (1000,1000) m and drone one is at (0,0). The uncertainty, σ is the standard deviation after 20,000 calculations. The uncertainty in all position and range measurements is 1.0 m.

In all conceivable scenarios, uncertainty will be present in every range and position measurement. Therefore, the accuracy of the solutions given in equation 9 and the uncertainty should be affected by the number of drones, n.

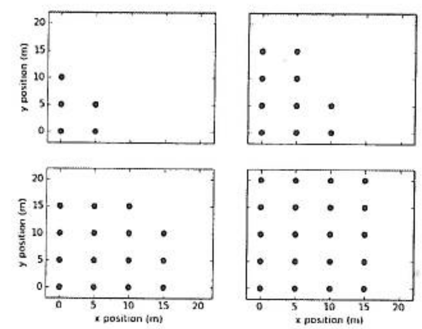
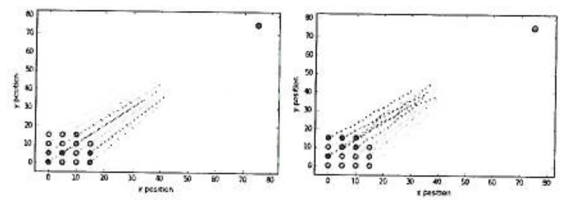


Figure 2: Starting positions for 5, 10, 15, and 20 drone swarms.

This result is shown in table 1, by assuming an uncertainty of 1.0 m and a goal beacon placed at a location of (1000, 1000) m. The swarm is distributed in a uniform, rectangular pattern, with the x-positions and y-positions spaced 5 m apart. Drone one is always placed at (0,0) and all drones start with positive coordinates. The starting configurations for each simulation are shown in figure 2. The data in the table represent the results after 20,000 simulations.

The asymmetry in the x and y standard deviation represents the asymmetry in the initial deployment position of the swarm with respect to the beacon. This can be seen most easily be examining the results for the fifteen-drone swarm, in which the ratio of the standard deviations in x and y is closest to one. This swarm is the most symmetric with regards to the direction to the goal, as seen in 2.

Figure 3 shows the first twenty time steps for a swarm of fifteen drones towards a goal beacon located at (75,75) m. Figure 3a depicts perfect range and position measurements and figure 3b depicts an uncertainty of 1.0 m in range and position measurements, simulated by gaussian noise centered on the actual position and with $\sigma = 1.0$ m.



(a) (b)

Figure 3: Simulation results for a swarm of 15 drones. Figure 3a has zero uncertainty in range and position measurements; Figure 3b has uncertainty in range and position measurements of 1.0 m

2.1. A simple range-finder model

Obtaining a measurement of range from a given drone to the goal beacon is not a simple problem, particularly if a system designer wishes to avoid a

transponder for the task. Most range finding systems, such as Distance Measuring Equipment (DME) used on aircraft require a mutual exchange of information between the goal beacon and the aerial vehicle.

One scenario for determining distance to an object of unknown location is to consider a transmitter of known power, a receiver, and a solution based on an assumed attenuation model. For free space propagation, we can write an equation for loss as

$$\mathrm{L}(dB) = 10\log\frac{P_t}{P_r} = 10\log\left(\frac{4\pi d}{c/f}\right)^2 \tag{10}$$

where P_t is the transmitted power, P_r is the receiver power, f is the frequency of the transmitted signal, c is the speed of light, and d is the distance between the receiver and transmitter.

An alternative expression, one that includes the effects of scattering and other attenuation, is given as

$$L(db) = 10\log\frac{P_t}{P_r} = 40 + 10n\log d + L_{other} \tag{11}$$

where n is a term that captures scattering effects, L_{other} captures other losses in the path, d is measured in km, and the frequency is assumed to be 2.45 GHz.

The values for n and L_{other} are scenario dependent [5]. For typical scenarios, the scattering coefficient n can range from 2 (representing free space propagation) to 4 (representing a heavily wooded area), and L_{other} can range from 0 (free space) to 30 (heavily wooded area). Using these assumptions about scattering, attenuation, and transmitted power, a receiver can find a distance by measuring received power.

It should be noted that I am not making any claims on the practicality of implementing the above model for range determination. It is my goal to show that the approach discussed in the previous sections is robust enough to handle incomplete or inaccurate range calculations using this generalized model.

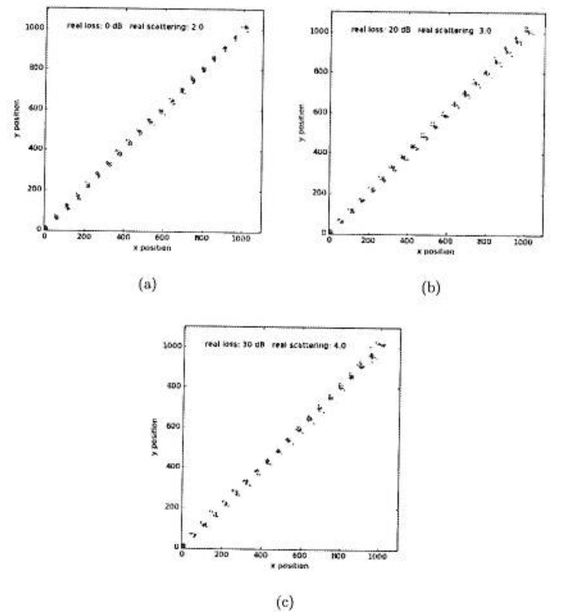


Figure 4: Simulation results for a swarm of 15 drones. Range to the beacon is determined based on assumptions in signal attenuation. Figure 4a shows the correct assumption for both attenuation and scattering; Figure 4b assumes free space propagation when scattering and attenuation are actually through lightly wooded areas; Figure 4c assumes free space propagation when scattering and attenuation are actually through heavily wooded areas.

3. Simulations and Results

The progression of a fifteen-drone swarm was calculated for several scenarios in which the wrong assumptions were made by the autonomous vehicles, e.g. the vehicles assumed a free space propagation instead of a heavily wooded area. The results are shown in figure 4.

In each case, the swarms converged on the beacon. As L_{other}, the constant loss term, increased, the swarm showed signs of dispersing as it approached the goal. The important observation, however, is the swarm still converged. The dispersion is understandable, as the constant loss term will cause the swarm to think the beacon is farther away than it actually is, even when the swarm has reached the goal.

3.1. Discussion

The work in the previous sections shows the robustness of the algorithm, but many technical issues have not been directly addressed.

Integration of sensor data. With the assumption of a GPS-denied environment, position (as well as pose) is determined through integration of commands and augmented by data from on-board sensors such as accelerometers and gyroscopes. If not executed well, this approach can quickly lead to unacceptable and increasing errors in position. More advanced techniques, such as using on-board cameras, are addressed in other works [4] [3] and will not be discussed here. I have assumed that the drone is capable of maintaining positional data with the accuracy stated for each simulation.

Minimal spacing. The algorithm I described included no mechanism for ensuring the swarm does not converge to a single point. That safeguard, as with all other details of autonomous navigation, were assumed to be a part of a baseline control algorithm. A constrained least squares approach was considered for handling issues such as spacing, but was rejected to minimize complexity in the algorithm.

4. Conclusion

The algorithm presented in this paper demonstrates an important capability available though the use of multiple, autonomous drones: the ability to use multiple entities, each possessing limited data, to solve a complex problem. Though this approach was conceived with quadrotors in mind, it certainly can be expanded to other autonomous vehicles and problems.

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